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$\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in universal extra dimensions

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Abstract. We study the exclusive $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in the Appelquist, Chang, Dobrescu model with a single universal extra dimension. We investigate the sensitivity of the branching ratio, lepton polarization and forward–backward asymmetry $\mathcal{A}_{\rm FB}$ to the compactification parameter $1/R$. We obtain the result that the branching ratio for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ $(\ell = \mu, \tau)$ decay changes about 25% compared to the SM value when $1/R = 250$ GeV, and the zero position of the forward–backward asymmetry is shifted to the left compared to the SM result. Therefore, measurement of the branching ratio of $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay and determination of the zero position of $\mathcal{A}_{\rm FB}$ are very useful in looking for new physics in the framework of the UED models.

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1 Introduction

Flavor-changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ transitions are forbidden in the standard model (SM) at tree level that occur at loop level, and therefore provide a consistency check of the SM at the quantum level. These decays induced by the FCNC are also very sensitive to the new physics beyond the SM. New physics is embedded in rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian (see [1] and references therein).

Among the hadronic, leptonic and semileptonic decays, the last decay channels are very significant, since they are theoretically more or less clean, and they have a relatively larger branching ratio. From the theoretical side there are many works in which the semileptonic decay channels due to $b \to s(d)\ell^+\ell^-$ transitions are investigated. These decays contain many observables like the forward–backward asymmetry $\mathcal{A}_{\rm FB}$, lepton polarization asymmetries, etc., which are very useful and serve as a testing ground for the SM and are suited for looking for new physics beyond the SM [1]. From the experimental side, the BELLE [2, 3] and BaBar [4, 5] collaborations provide recent measurements of the branching ratios of the semileptonic decays due to the $b \to s\ell^+\ell^-$ transitions, which can be summarized as follows:

$$\mathcal{B}(B \to K^* \ell^+ \ell^-) = \begin{cases} \left(16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4\right) \times 10^{-7} & [2]\,, \\ \left(7.8^{+1.9}_{-1.7} \pm 1.2\right) \times 10^{-7} & [4]\,, \end{cases}$$

$$\mathcal{B}(B \to K \ell^+ \ell^-) = \begin{cases} \left(5.5^{+0.75}_{-0.70} \pm 0.27 \pm 0.02\right) \times 10^{-7} & [2]\,, \\ (3.4 \pm 0.7 \pm 0.3) \times 10^{-7} & [4]\,. \end{cases}$$

$$\mathcal{B}(B \to X_s \ell^+ \ell^-) = \begin{cases} \left(4.11 \pm 0.83^{+0.85}_{-0.81}\right) \times 10^{-6} & [3]\,, \\ (5.6 \pm 1.5 \pm 0.6 \pm 1.1) \times 10^{-6} & [5]\,. \end{cases}$$

Another exclusive decay which is described at inclusive level by the $b \to s\ell^+\ell^-$ transition is the baryonic $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay. Unlike mesonic decays, the baryonic decays could maintain the helicity structure of the effective Hamiltonian for the $b \to s$ transition [6]. Radiative and semileptonic decays of Λ_b such as $\Lambda_b \to \Lambda\gamma$, $\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$, $\Lambda_b \to \Lambda \ell^+ \ell^-$ $(\ell = e, \mu, \tau)$ and $\Lambda_b \to \Lambda \nu \bar{\nu}$ have been extensively studied in the literature [7–34]. More about heavy baryons, including the experimental prospects, can be found in [7–21, 36].

Among the various models of physics beyond the SM, extra dimensions attract special interest, because they include gravity in addition to other interactions, giving hints on the hierarchy problem and a connection with string theory. The model of Appelquist, Cheng and Dobrescu (ACD) [37] with a single universal extra dimension (UED) [38], where all the SM particles can propagate in the extra dimension, are very attractive. Compactification of the extra dimension leads to a Kaluza–Klein (KK) model in dimension four. In this model the only additional free parameter with respect to the SM is $1/R$, i.e., the inverse of the compactification radius.

The restrictions imposed on UED are examined in the current accelerators; for example, Tevatron experiments put the bound about $1/R \geq 300$ GeV. Analysis of the anomalous magnetic moment [39, 40] and the $Z \to \bar{b}b$ vertex [41] also leads to the bound $1/R \geq 300$ GeV.

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A possible manifestation of UED models in the $K_{\rm L}$–$K_{\rm S}$ mass difference, parameter ε_K, B–$\bar{B}_0$ mixing, the $\Delta M_{d,s}$ mass difference, and the rare decays $K^+ \to \pi\bar{\nu}\nu$, $K_{\rm L} \to \pi^0\bar{\nu}\nu$, $K_{\rm L} \to \mu^+\mu^-$, $B \to X_{s,d}\bar{\nu}\nu$, $B_{s,d} \to \mu^+\mu^-$, $B \to X_s\gamma$, $B \to X_s\, g$, $B \to X_s\mu^+\mu^-$ and ε'/ε are comprehensively investigated in [42, 43]. Exclusive $B \to K^*\ell^+\ell^-$, $B \to K^*\bar{\nu}\nu$ and $B \to K^*\gamma$ decays are studied in the framework of the UED scenario in [44], and it is shown that the most stringent bound comes from the $B \to K^*\gamma$ decay, restricting R to have the value $1/R \geq 250$ GeV, which we will use in our numerical analysis.

In the present work we study the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in the UED model. The plan of the paper is as follows. In Sect. 2 we briefly discuss the main ingredients of the ACD model and study the rare $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in it. Section 3 is devoted to the numerical analysis and conclusions.

2 Theoretical background for the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in a universal extra dimension model

Before presenting a detailed derivation of the matrix element of $\Lambda_b \to \Lambda\ell^+\ell^-$ decay, let us discuss the main ingredients of the ACD model, which is the minimal extension of the SM in $4+\delta$ dimensions, and we consider the simplest case, $\delta = 1$. The five-dimensional ACD model with a single UED uses orbifold compactification. The fifth dimension that is compactified in a circle of radius R, with points $y = 0$ and $y = \pi R$, that are fixed points of the orbifolds. Generalization to the SM is realized by the propagating fermions, gauge bosons and the Higgs fields in all five dimensions. The Lagrangian can be written as

$$\mathcal{L} = \int {\rm d}^4x{\rm d}y \Big\{ \mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y \Big\} ,$$

where

$$\begin{aligned}
\mathcal{L}_A &= -\frac{1}{4} W^{MNa} W^a_{MN} - \frac{1}{4} B^{MN} B_{MN} ,\\
\mathcal{L}_H &= \Big(\mathcal{D}^M\phi\Big)^\dagger \mathcal{D}_M\phi - V(\phi) ,\\
\mathcal{L}_F &= \bar{\mathcal{Q}}\Big({\rm i}\Gamma^M\mathcal{D}_M\Big)\mathcal{Q} + \bar{u}\Big({\rm i}\Gamma^M\mathcal{D}_M\Big)u + \bar{\mathcal{D}}\Big({\rm i}\Gamma^M\mathcal{D}_M\Big)\mathcal{D} ,\\
\mathcal{L}_Y &= -\bar{\mathcal{Q}}\widetilde{Y}_u\phi^c u - \bar{\mathcal{Q}}\widetilde{Y}_d\phi\mathcal{D} + {\rm h.c.}
\end{aligned}$$

Here M and N, running over $0, 1, 2, 3, 5$, are the five-dimensional Lorentz indices, $W^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + \tilde{g}\varepsilon^{abc}W^b_M W^c_N$ is the field strength tensor for the $SU(2)_{\rm L}$ electroweak gauge group, $B_{MN} = \partial_M B_N - \partial_N B_M$ is that of the $U(1)$ group, and all fields depend both on x and y. The covariant derivative is defined as $\mathcal{D}_M = \partial_M - {\rm i}\tilde{g}W^a_M\, T^a - {\rm i}\tilde{g}'B_M Y$, where $\tilde{g}$ and $\tilde{g}'$ are the five-dimensional gauge couplings for the $SU(2)_{\rm L}$ and $U(1)$ groups. The five-dimensional Γ_M matrices are defined as $\Gamma^\mu = \gamma^\mu$, $\mu = 0, 1, 2, 3$ and $\Gamma^5 = {\rm i}\gamma^5$.

In the case of a single extra dimension with coordinate $x_5 = y$ compactified on a circle of radius R, a field $F(x, y)$ would be a periodic function of y, and hence can be written as

$$F(x, y) = \sum_{n=-\infty}^{+\infty} F_n(x) e^{{\rm i}ny/R} .$$

The Fourier expansion of the fields is

$$\begin{aligned}
B_\mu(x, y) &= \frac{1}{\sqrt{2\pi R}} B^{(0)}_\mu + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B^{(0)}_\mu(x) \cos\left(\frac{ny}{R}\right) ,\\
B_5(x, y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B^{(n)}_5 \sin\left(\frac{ny}{R}\right) ,\\
\mathcal{Q}(x, y) &= \frac{1}{\sqrt{2\pi R}} \mathcal{Q}^{(0)}_{\rm L} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \Big[\mathcal{Q}^{(n)}_{\rm L} \cos\left(\frac{ny}{R}\right)\\
&\quad + \mathcal{Q}^{(n)}_{\rm R} \sin\left(\frac{ny}{R}\right)\Big] ,\\
U(\mathcal{D})(x, y) &= \frac{1}{\sqrt{2\pi R}} U^{(0)}_{\rm R} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \Big[U^{(n)}_{\rm R} \cos\left(\frac{ny}{R}\right)\\
&\quad + U^{(n)}_{\rm L} \sin\left(\frac{ny}{R}\right)\Big] .
\end{aligned}$$

Under a parity transformation $P_5 : y \to -y$ fields having a corresponding one in the four-dimensional SM it should be even, so that their zero-modes in the KK can be interpreted as the ordinary SM field. Fields having no corresponding one in the SM should be odd. From this expansion we see that the fifth component of the vector field is odd under a P_5 transformation.

One important property of the ACD model is that the KK parity is conserved. The parity conservation leads to the result that there is no tree level contribution of the KK modes in low energy processes (at the scale $\mu \ll 1/R$) and a single KK excitation cannot be produced in an ordinary particle interaction. Finally note that in the ACD model there are three additional physical scalar modes: $a^{(0)}_n$ and $a^\pm_n$.

The zero mode is either right-handed or left-handed. The nonzero modes come in a chiral pair. This chirality is a consequence of the orbifold boundary conditions.

The Lagrangian of the ACD model can be obtained by integrating over $x_5 = y$:

$$\mathcal{L}_4(x) = \int_0^{2\pi R} \mathcal{L}_5(x, y){\rm d}y .$$

Note that the zero mode remains massless unless we apply the Higgs mechanism. All fields in the four-dimensional Lagrangian receive the KK mass n/R on account of the derivative operator ∂_5 acting on them. The relevant Feynman rules are derived in [42] and for more details about the ACD model we refer the interested reader to [42, 43].

After this preliminary introduction, let us discuss the main problem. In this section we present the matrix element of $\Lambda_b \to \Lambda\ell^+\ell^-$ decay as well as expressions of the branching ratio, forward–backward asymmetry and lepton polarizations.

At the quark level, $\Lambda_b \to \Lambda\ell^+\ell^-$ decay is described by a $b \to s\ell^+\ell^-$ transition. The effective Hamiltonian governing this transition in the SM with $\Delta B = -1$, $\Delta S = 1$ is described in terms of a set of local operators:

$$\mathcal{H}_{\rm eff} = \frac{4G_{\rm F}}{\sqrt{2}} V_{tb} V_{ts}^* \sum_1^{10} C_i(\mu) \mathcal{O}_i(\mu) , \qquad (1)$$

where $G_{\rm F}$ is the Fermi constant, and V_{ij} are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Explicit forms of the operators, which are written in terms of quark and gluon fields and the corresponding Wilson coefficients in (1), which are computed at NNLO in the SM, can be found in [45–47]. At NLO these Wilson coefficients are calculated for the ACD model including the effects of KK modes [42, 43], which we have used in our calculations. It should be noted here that there does not appear any new operator in the ACD model, and, therefore, new effects are implemented by modifying the Wilson coefficients existing in the SM, if we neglect the contributions of the scalar fields, which are indeed very small.

At $\mu = \mathcal{O}(m_W)$ level, only $C_2^{(0)}$, $C_7^{(0)}(m_W)$, $C_8^{(0)}(m_W)$, $C_9^{(0)}(m_W)$ and $C_{10}^{(0)}(m_W)$ are different from zero, and the remaining coefficients are all zero.

In the following we do not consider the contribution to $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay from the lepton pair being created from a $\bar{c}c$ resonance due to the $\mathcal{O}_2$ operators. It can be removed by applying appropriate cuts to the invariant dilepton mass around the mass of the resonance.

The matrix element of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described with the help of the Wilson coefficients C_7, C_9 and C_{10} as follows:

$$\begin{aligned}\mathcal{M} = \frac{G_{\rm F}}{4\sqrt{2}} V_{tb} V_{ts}^* \Big\{ & C_9^{\rm SM} \bar{s} \gamma^\mu (1-\gamma_5) b \bar{\ell} \gamma_\mu \ell \\ & + C_{10}^{\rm SM} \bar{s} \gamma^\mu (1-\gamma_5) b \bar{\ell} \gamma_\mu \gamma_5 \ell \\ & - 2 m_b C_7^{\rm SM} \bar{s} \mathrm{i} \sigma_{\mu\nu} \frac{q^\nu}{q^2} (1+\gamma_5) b \bar{\ell} \gamma^\mu \ell \Big\} . \end{aligned} \qquad (2)$$

The Wilson coefficients in the ACD model receive new contributions, since new particles which are absent in the SM can contribute as intermediate states in penguin and box diagrams. As a result, the Wilson coefficients can be expressed in terms of the functions $f(x_t, 1/R)$, which modify the corresponding SM functions $f_0(x_t)$ according to

$$f(x_t, 1/R) = f_0(x_t) + \sum_{n=1}^{\infty} f_n(x_t, x_n) , \qquad (3)$$

where $x_t = m_t^2/m_W^2$, $x_n = m_n^2/m_W^2$ and $m_n = n/R$.

Contributions to these Wilson coefficients coming from the new particles existing in UED are calculated in [42–44] they can be written as

$$\begin{aligned} C_7^{(0)}(m_W) &= -\frac{1}{2} D'(x_t, 1/R) , \\ C_8^{(0)}(m_W) &= -\frac{1}{2} E'(x_t, 1/R) , \\ C_9^{(0)}(m_W) &= P_0^{NDR} + \frac{Y(x_t, 1/R)}{\sin^2\theta_{\rm W}} - 4Z(x_t, 1/R) \\ &\quad + P_E E(x_t, 1/R) , \\ C_{10}^{(0)} &= -\frac{Y(x_t, 1/R)}{\sin^2\theta_{\rm W}} . \end{aligned} \qquad (4)$$

Explicit expressions of P_0^{NDR} and P_E can be found in [45–51], and numerically $P_0^{NDR} = 2.60 \pm 0.25$, and P_E is of the order of $\mathcal{O}(10^{-2})$, so that the last term of $C_9^{(0)}(m_W)$ in (4) can be neglected. Here the superscript (0) refers to the leading log approximation.

The functions D' and E', which describe electromagnetic and chromomagnetic penguins, respectively, are calculated in [42, 43] and lead to the following results [44]:

$$D_0'(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4} \ln x_t , \qquad (5)$$

$$E_0'(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^4} \ln x_t , \qquad (6)$$

$$\begin{aligned} \sum_{n=1}^{\infty} & D_n'(x_t, x_n) \\ = & \frac{x_t[37 - x_t(44 + 17x_t)]}{72(x_t-1)^3} \\ & + \frac{\pi m_W R}{12} \left[\int_0^1 \mathrm{d}y (2y^{1/2} + 7y^{3/2} + 3y^{5/2}) \right. \\ & \times \coth(\pi m_W R \sqrt{y}) \\ & - \frac{x_t(2-3x_t)(1+3x_t)}{(x_t-1)^4} J(R, -1/2) \\ & - \frac{1}{(x_t-1)^4} \{ x_t(1+3x_t) + (2-3x_t)[1 - (10-x_t)x_t] \} \\ & \times J(R, 1/2) \\ & - \frac{1}{(x_t-1)^4} [(2-3x_t)(3+x_t) + 1 - (10-x_t)x_t] J(R, 3/2) \\ & \left. - \frac{(3+x_t)}{(x_t-1)^4} J(R, 5/2) \right] , \end{aligned} \qquad (7)$$

$$\begin{aligned} \sum_{n=1}^{\infty} & E_n'(x_t, x_n) \\ = & \frac{x_t[17 + (8-x_t)x_t]}{24(x_t-1)^3} \\ & + \frac{\pi m_W R}{4} \left[\int_0^1 \mathrm{d}y (y^{1/2} + 2y^{3/2} - 3y^{5/2}) \right. \\ & \times \coth(\pi m_W R \sqrt{y}) \\ & - \frac{x_t(1+3x_t)}{(x_t-1)^4} J(R, -1/2) \\ & + \frac{1}{(x_t-1)^4} [x_t(1+3x_t) - 1 + (10-x_t)x_t] J(R, 1/2) \\ & - \frac{1}{(x_t-1)^4} [(3+x_t) - 1 + (10-x_t)x_t)] J(R, 3/2) \\ & \left. + \frac{(3+x_t)}{(x_t-1)^4} J(R, 5/2) \right] , \end{aligned} \qquad (8)$$

where

$$J(R,\alpha) = \int_0^1 \mathrm{d}y y^\alpha \left[\coth(\pi m_W R\sqrt{y}) - x_t^{1+\alpha}\coth(\pi m_t R\sqrt{y})\right] \ . \tag{9}$$

The functions $Y(x_t, 1/R)$ and $Z(x_t, 1/R)$ are defined by

$$Y(x_t,1/R) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t,x_n) \ ,$$
$$Z(x_t,1/R) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t,x_n) \ , \tag{10}$$

with

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-4}{x_t-1} + \frac{3x_t}{(x_t-1)^2}\ln x_t\right] \ , \tag{11}$$

$$Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \left[\frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t-1)^4} - \frac{1}{9}\right]\ln x_t \tag{12}$$

$$\sum_{n=1}^{\infty} C_n(x_t,x_n) = \frac{x_t(7-x_t)}{16(x_t-1)} - \frac{\pi m_W R x_t}{16(x_t-1)^2}\left[3(1+x_t)J(R,-1/2) + (x_t-7)J(R,1/2)\right] \ . \tag{13}$$

With the help of these Wilson coefficients and the matrix element given in (2), the inclusive $b \to s\ell^+\ell^-$ transitions have been studied in [42, 43].

The amplitude of the exclusive $\Lambda_b \to \Lambda\ell^+\ell^-$ decay is obtained by sandwiching the matrix element of the $b \to s\ell^+\ell^-$ transition between initial and final baryon states $\langle \Lambda |\mathcal{M}| \Lambda_b\rangle$. It follows from (2) that the matrix elements

$$\langle \Lambda |\bar{s}\gamma_\mu(1-\gamma_5)b| \Lambda_b\rangle \ , \tag{14}$$
$$\langle \Lambda |\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b| \Lambda_b\rangle \tag{15}$$

are needed in order to calculate the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay amplitude.

These matrix elements are parametrized in terms of the form factors as follows [32, 52]:

$$\langle \Lambda |\bar{s}\gamma_\mu b| \Lambda_b\rangle = \bar{u}_\Lambda\left[f_1\gamma_\mu + \mathrm{i}f_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu\right]u_{\Lambda_b} \ , \tag{16}$$

$$\langle \Lambda |\bar{s}\gamma_\mu\gamma_5 b| \Lambda_b\rangle = \bar{u}_\Lambda\left[g_1\gamma_\mu\gamma_5 + \mathrm{i}g_2\sigma_{\mu\nu}\gamma_5 q^\nu + g_3 q_\mu\gamma_5\right]u_{\Lambda_b} \ , \tag{17}$$

where $q = p_{\Lambda_b} - p_\Lambda$.

The form factors of the magnetic dipole operators are defined as

$$\langle \Lambda |\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu b| \Lambda_b\rangle = \bar{u}_\Lambda\left[f_1^{\mathrm{T}}\gamma_\mu + \mathrm{i}f_2^{\mathrm{T}}\sigma_{\mu\nu}q^\nu + f_3^{\mathrm{T}}q_\mu\right]u_{\Lambda_b} \ ,$$
$$\langle \Lambda |\bar{s}\mathrm{i}\sigma_{\mu\nu}\gamma_5 q^\nu b| \Lambda_b\rangle = \bar{u}_\Lambda\left[g_1^{\mathrm{T}}\gamma_\mu\gamma_5 + \mathrm{i}g_2^{\mathrm{T}}\sigma_{\mu\nu}\gamma_5 q^\nu + g_3^{\mathrm{T}}q_\mu\gamma_5\right]u_{\Lambda_b} \ . \tag{18}$$

Using the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{\mathrm{i}}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \ ,$$

the following relations between the form factors are obtained:

$$f_1^{\mathrm{T}} = -\frac{q^2}{m_{\Lambda_b} - m_\Lambda}f_3^{\mathrm{T}} \ ,$$
$$g_1^{\mathrm{T}} = \frac{q^2}{m_{\Lambda_b} + m_\Lambda}g_3^{\mathrm{T}} \ . \tag{19}$$

Using these definitions of the form factors, for the matrix element of the $\Lambda_b \to \Lambda\ell^+\ell^-$ we get

$$\begin{aligned}\mathcal{M} = \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^*\frac{1}{2}\Bigg\{&\bar{\ell}\gamma_\mu(1-\gamma_5)\ell \\ &\times \bar{u}_\Lambda\Big[(A_1-D_1)\gamma_\mu(1+\gamma_5) + (B_1-E_1)\gamma_\mu(1-\gamma_5) \\ &+ \mathrm{i}\sigma_{\mu\nu}q^\nu\Big((A_2-D_2)(1+\gamma_5) + (B_2-E_2)(1-\gamma_5)\Big) \\ &+ q_\mu\Big((A_3-D_3)(1+\gamma_5) + (B_3-E_3)(1-\gamma_5)\Big)\Big]u_{\Lambda_b} \\ &+ \bar{\ell}\gamma_\mu(1+\gamma_5)\ell\,\bar{u}_\Lambda\Big[(A_1+D_1)\gamma_\mu(1+\gamma_5) \\ &+ (B_1+E_1)\gamma_\mu(1-\gamma_5) \\ &+ \mathrm{i}\sigma_{\mu\nu}q^\nu\Big((A_2+D_2)(1+\gamma_5) + (B_2+E_2)(1-\gamma_5)\Big) \\ &+ q_\mu\Big((A_3+D_3)(1+\gamma_5) + (B_3+E_3)(1-\gamma_5)\Big)\Big]u_{\Lambda_b}\Bigg\} \ ,\end{aligned} \tag{20}$$

where

$$\begin{aligned} A_1 &= \frac{1}{q^2}\left(f_1^{\mathrm{T}} - g_1^{\mathrm{T}}\right)(-2m_s C_7) \\ &\quad + \frac{1}{q^2}\left(f_1^{\mathrm{T}} + g_1^{\mathrm{T}}\right)(-2m_b C_7) + (f_1 - g_1)\,C_9^{\mathrm{eff}} \ , \\ A_2 &= A_1\,(1\to 2) \ , \\ A_3 &= A_1\,(1\to 3) \ , \\ B_1 &= A_1\left(g_1 \to -g_1;\ g_1^{\mathrm{T}} \to -g_1^{\mathrm{T}}\right) \ , \\ B_2 &= B_1\,(1\to 2) \ , \\ B_3 &= B_1\,(1\to 3) \ , \\ D_1 &= C_{10}\,(f_1 - g_1) \ , \\ D_2 &= D_1\,(1\to 2) \ , \\ D_3 &= D_1\,(1\to 3) \ , \\ E_1 &= D_1\,(g_1 \to -g_1) \ , \\ E_2 &= E_1\,(1\to 2) \ , \\ E_3 &= E_1\,(1\to 3) \ . \end{aligned} \tag{21}$$

From these expressions it follows that $\Lambda_b \to \Lambda\ell^+\ell^-$ decay is described in terms of many form factors. It is shown in [6, 53] that heavy quark effective theory (HQET) reduces the number of independent form factors to two (F_1

and F_2), independent of the Dirac structure of the corresponding operators, i.e.,

$$\left\langle \Lambda(p_\Lambda) \left| \bar{s} \Gamma b \right| \Lambda(p_{\Lambda_b}) \right\rangle = \bar{u}_\Lambda \Big[F_1(q^2) + \not{v} F_2(q^2) \Big] \Gamma u_{\Lambda_b} , \tag{22}$$

where Γ is an arbitrary Dirac structure and $v^\mu = p^\mu_{\Lambda_b}/m_{\Lambda_b}$ is the four-velocity of Λ_b. Comparing the general form of the form factors given in (16)–(18) with (22), one can easily obtain the following relations among them [32, 33, 52]

$$\begin{aligned}
g_1 &= f_1 = f_2^{\rm T} = g_2^{\rm T} = F_1 + \sqrt{\hat{r}_\Lambda} F_2 , \\
g_2 &= f_2 = g_3 = f_3 = \frac{F_2}{m_{\Lambda_b}} , \\
g_1^{\rm T} &= f_1^{\rm T} = \frac{F_2}{m_{\Lambda_b}} q^2 , \\
g_3^{\rm T} &= \frac{F_2}{m_{\Lambda_b}} \left(m_{\Lambda_b} + m_\Lambda \right) , \\
f_3^{\rm T} &= -\frac{F_2}{m_{\Lambda_b}} \left(m_{\Lambda_b} - m_\Lambda \right) ,
\end{aligned} \tag{23}$$

where $\hat{r}_\Lambda = m_\Lambda^2 / m_{\Lambda_b}^2$.

In what follows, we will be looking for the possible manifestation of UED theory in the branching ratio as well as in lepton polarizations. For this purpose, we present the decay rate for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ taking into account lepton polarizations.

Let us define the following orthogonal unit vectors: $s^{-\mu}$ in the rest frame of the lepton and $s^{+\mu}$ in the rest frame of the antilepton for the polarization of the leptons along the longitudinal (L) direction, the normal (N) direction and the transversal (T) direction of the lepton and antilepton momentum vector, respectively:

$$\begin{aligned}
s_{\rm L}^{-\mu} &= \left(0, \mathbf{e}_{\rm L}^-\right) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right) , \\
s_N^{-\mu} &= \left(0, \mathbf{e}_{\rm N}^-\right) = \left(0, \frac{\mathbf{p}_\Lambda \times \mathbf{p}_-}{|\mathbf{p}_\Lambda \times \mathbf{p}_-|}\right) , \\
s_{\rm T}^{-\mu} &= \left(0, \mathbf{e}_{\rm T}^-\right) = \left(0, \mathbf{e}_{\rm N}^- \times \mathbf{e}_{\rm L}^-\right) , \\
s_{\rm L}^{+\mu} &= \left(0, \mathbf{e}_{\rm L}^+\right) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right) , \\
s_{\rm N}^{+\mu} &= \left(0, \mathbf{e}_{\rm N}^+\right) = \left(0, \frac{\mathbf{p}_\Lambda \times \mathbf{p}_+}{|\mathbf{p}_\Lambda \times \mathbf{p}_+|}\right) , \\
s_{\rm T}^{+\mu} &= \left(0, \mathbf{e}_{\rm T}^+\right) = \left(0, \mathbf{e}_{\rm N}^+ \times \mathbf{e}_{\rm L}^+\right) ,
\end{aligned} \tag{24}$$

where $\mathbf{p}_-(\mathbf{p}_+)$ and $\mathbf{p}_\Lambda$ are the three-momenta of the leptons $\ell^-(\ell^+)$ and the Λ baryon in the center of mass frame (CM) of the $\ell^-\ell^+$ system, respectively.

The longitudinal component of the lepton (antilepton) polarization is boosted to the CM frame of the lepton pair by a Lorentz transformation, yielding

$$\begin{aligned}
\left(s_{\rm L}^{-\mu}\right)_{CM} &= \left(\frac{|\mathbf{p}_-|}{m_\ell} , \frac{E_\ell \, \mathbf{p}_-}{m_\ell \, |\mathbf{p}_-|} \right) \\
\left(s_{\rm L}^{+\mu}\right)_{CM} &= \left(\frac{|\mathbf{p}_-|}{m_\ell} , -\frac{E_\ell \, \mathbf{p}_-}{m_\ell \, |\mathbf{p}_-|} \right) ,
\end{aligned} \tag{25}$$

where E_ℓ and m_ℓ are the energy and mass of ℓ^- in the CM frame. The remaining two unit vectors $s_{\rm N}^{\pm\mu}$, $s_{\rm T}^{\pm\mu}$ are unchanged under a Lorentz boost.

The differential decay rate for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay along any spin direction $\xi^{\,\pm}$ along the momentum vector of $\ell^\pm$ can be written as

$$\frac{{\rm d}\Gamma(\xi^\mp)}{{\rm d}\hat{s}} = \frac{1}{2} \left(\frac{{\rm d}\Gamma}{{\rm d}\hat{s}} \right)_0 \times \left[1 + \left(P_{\rm L}^\mp \mathbf{e}_{\rm L}^\mp + P_{\rm N}^\mp \mathbf{e}_{\rm N}^\mp + P_{\rm T}^\mp \mathbf{e}_{\rm T}^\mp \right) \cdot \xi^{\,\mp} \right] , \tag{26}$$

where $({\rm d}\Gamma/{\rm d}\hat{s})_0$ corresponds to the unpolarized differential decay rate, $\hat{s} = q^2/m_{\Lambda_b}^2$ and $P_{\rm L}$, $P_{\rm N}$ and $P_{\rm T}$ represent the longitudinal, normal and transversal polarizations of ℓ, respectively, and it has the following form:

$$\left(\frac{{\rm d}\Gamma}{{\rm d}\hat{s}} \right)_0 = \frac{G^2 \alpha^2}{8192 \pi^5} \left| V_{tb} V_{ts}^* \right|^2 \lambda^{1/2}(1, r, \hat{s}) v \Big[\mathcal{T}_0(\hat{s}) + \frac{1}{3} \mathcal{T}_2(\hat{s}) \Big] , \tag{27}$$

where $\lambda(1, r, \hat{s}) = 1 + r^2 + \hat{s}^2 - 2r - 2\hat{s} - 2r\hat{s}$ is the triangle function and $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity.

The polarizations $P_{\rm L}$, $P_{\rm N}$ and $P_{\rm T}$ are defined by

$$P_i^{(\mp)}(\hat{s}) = \frac{ \dfrac{{\rm d}\Gamma}{{\rm d}\hat{s}}(\xi^{\,\mp} = \mathbf{e}_i^\mp) - \dfrac{{\rm d}\Gamma}{{\rm d}\hat{s}}(\xi^{\,\mp} = -\mathbf{e}_i^\mp) }{ \dfrac{{\rm d}\Gamma}{{\rm d}\hat{s}}(\xi^{\,\mp} = \mathbf{e}_i^\mp) + \dfrac{{\rm d}\Gamma}{{\rm d}\hat{s}}(\xi^{\,\mp} = -\mathbf{e}_i^\mp) } ,$$

where $i =$ L,N,T.

One of the efficient tools for establishing new physics effects is the study of the forward–backward asymmetry $\mathcal{A}_{\rm FB}$ which is defined as

$$\mathcal{A}_{\rm FB} = \frac{ \displaystyle\int_0^1 \frac{{\rm d}\Gamma}{{\rm d}\hat{s}{\rm d}z} \, {\rm d}z - \int_{-1}^0 \frac{{\rm d}\Gamma}{{\rm d}\hat{s}{\rm d}z} \, {\rm d}z }{ \displaystyle\int_0^1 \frac{{\rm d}\Gamma}{{\rm d}\hat{s}{\rm d}z} \, {\rm d}z + \int_{-1}^0 \frac{{\rm d}\Gamma}{{\rm d}\hat{s}{\rm d}z} \, {\rm d}z } ,$$

where the $z = \cos\theta$ dependence of the differential decay rate can be implemented by making the replacement

$$\mathcal{T}_0(\hat{s}) + \frac{1}{3} \mathcal{T}_2(\hat{s}) \to \mathcal{T}_0(\hat{s}) + \mathcal{T}_1(\hat{s}) z + \mathcal{T}_2(\hat{s}) z^2 ,$$

on the right-hand side and including $d\cos\theta$ in the denominator on the left-hand side of (27), where θ is the angle between Λ_b and ℓ^- in the CM of leptons. Explicit expressions of $\mathcal{T}_0(\hat{s})$, $\mathcal{T}_1(\hat{s})$ and $\mathcal{T}_2(\hat{s})$ can be found in [32].

It is well known that in the $B \to K^* \ell^+ \ell^-$ decay the zero position of $\mathcal{A}_{\rm FB}$ is practically independent of the form factors. For this reason, determination of the zero position of $\mathcal{A}_{\rm FB}$, as well as its magnitude, is very promising in looking for new physics beyond the SM. Note also that the combined analysis of the lepton polarizations can give additional information about the existence of new physics, since in the SM $P_{\rm L}^+ + P_{\rm L}^- = 0$, $P_{\rm N}^+ + P_{\rm N}^- = 0$ and $P_{\rm T}^+ - P_{\rm T}^- \approx 0$ (in the $m_\ell \to 0$ limit). Therefore any nonzero value resulting from these combined polarizations can be considered as a confirmation of new physics.

3 Numerical analysis

In this section we present our numerical results for the polarization asymmetries $P_{\rm L}$, $P_{\rm N}$ and $P_{\rm T}$ when one of the leptons is polarized. The values of the input parameters we use in our calculations are $|V_{tb}V_{ts}^*| = 0.0385$, $m_\tau = 1.77$ GeV, $m_\mu = 0.106$ GeV, $m_{\Lambda_b} = 5.62$ GeV, $m_b = 4.8$ GeV [54], $m_t = 172.7$ GeV [55] and $\tau_{\Lambda_b} = 1.23$ ps.

From the expressions of the asymmetries it follows that the form factors are the main and the most important input parameters necessary in the numerical calculations. The calculation of the form factors of $\Lambda_b \to \Lambda$ transition does not exist at present. But we can use the results from the QCD sum rules in co-operation with HQET [53, 56]. We noted earlier that HQET allows us to establish relations among the form factors and reduce the number of independent form factors to two. In [53, 56], the q^2 dependence of these form factors is given as follows:

$$F(\hat{s}) = \frac{F(0)}{1 + a_F \hat{s} + b_F \hat{s}^2}\,, \quad \text{set I.}$$

The values of the parameters $F(0)$, a_F and b_F are given in Table 1.

Note that the first analysis of the HQET structure of the $\Lambda_Q \to \Lambda_q$ transition is performed in [57, 58].

In order to get an idea of the sensitivity of our results to the specific parametrization of the two form factors predicted by the QCD sum rules in co-operation with the HQET, we also have used another parametrization of the form factors based on the pole model and compared the results of both models. The second set of form factors which have the dipole form predicted by the pole model are

Table 1. Form factors for $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in a three parameter fit

	$F(0)$	a_F	b_F
F_1	0.462	−0.0182	−0.000176
F_2	−0.077	−0.0685	0.00146

given by

$$F_{1,2}(E_\Lambda) = N_{1,2}\left(\frac{\Lambda_{\rm QCD}}{\Lambda_{\rm QCD} + E_\Lambda}\right)^2\,, \quad \text{set II,}$$

where

$$E_\Lambda = \frac{m_{\Lambda_b}^2 - m_\Lambda^2 - q^2}{2 m_{\Lambda_b}}\,,$$

and $\Lambda_{\rm QCD} = 0.2$, $N_1 = 52.32$ and $N_2 \simeq -0.25 N_1$ [59].

In Figs. 1 and 2 we present the dependence of the branching ratio for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ and $\Lambda_b \to \Lambda \tau^+ \tau^-$ decays on the compactification parameter $1/R$ for the set I of form factors, respectively. For completeness, we also present the SM prediction in these figures. In both cases we take into account the errors in the form factors. The errors inherent in $F_1(0)$ and $F_2(0)$ are estimated to be ± 0.03 for both of the form factors within the QCD sum rules method in [56]. Upper (lower) lines in these figures correspond to the case when errors in the form factors are added to (subtracted from) their central values. The analytical expressions of UED and SM are the same (except the values of the Wilson coefficients); errors in the form factors in both theories act in the same manner, i.e., if we add (subtract) the errors to the central values of the form factors in the SM, we should do the same in UED. Therefore, even if we take the errors into consideration in the form factors, the difference between the predictions of the two theories still remains the same. From

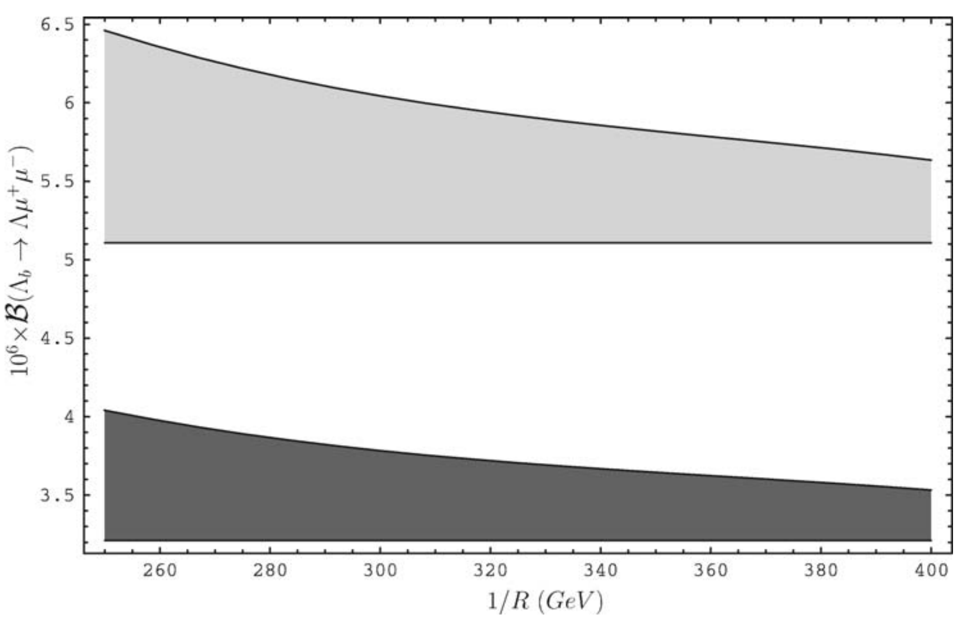

Fig. 1. The dependence of the branching ratio for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay on the compactification parameter $1/R$, for set I of the form factors. Note that the *upper* (*lower*) *two lines* correspond to the case when errors in the form factors are added to (subtracted from) their central values; the *curved lines* are predicted by the UED model and the *straight* ones by the SM

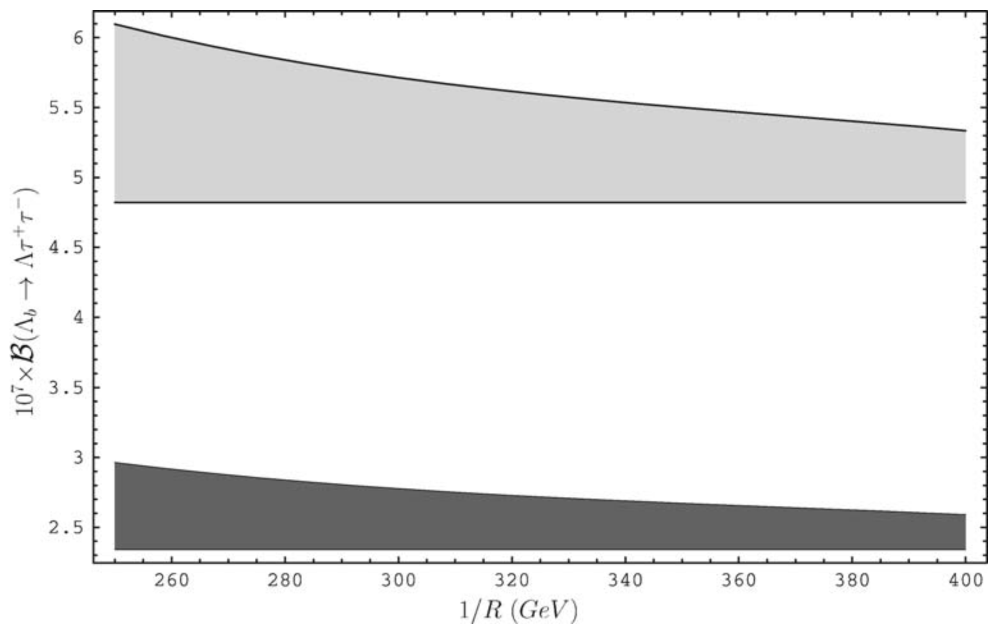

Fig. 2. The same as Fig. 1, but for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay

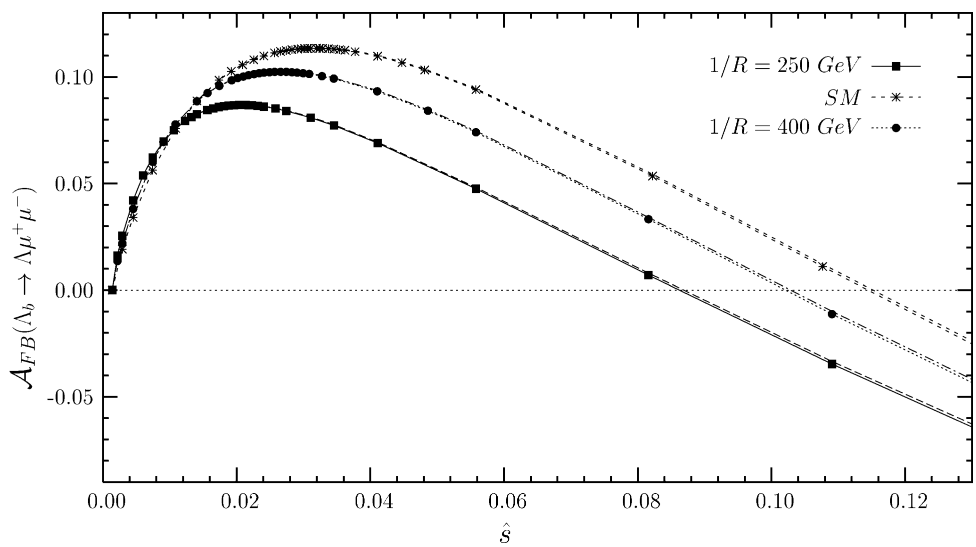

Fig. 3. The dependence of the forward–backward asymmetry $\mathcal{A}_{\rm FB}$ on $\hat{s}$ at two fixed values of $1/R$ and in the SM, for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay, for set I of the form factors. Note that each line corresponds to the case when errors in the form factors are added to (subtracted from) their central values. The values of $\mathcal{A}_{\rm FB}$ in these cases almost coincide with each other

these figures we see that, for both channels, the difference between the two models is around 20%–25%.

We have performed a similar analysis for set II of the form factors and obtained the result that the dependence of the differential branching ratio on the compactification parameter $1/R$ for both sets of form factors practically coincides and for this reason we do not present the results for set II of the form factors.

Analysis of the lepton polarizations leads to the following results.

- The maximum value of the difference between the SM and ADC model (for the minimum value of $1/R = 250$ GeV) predictions, as far as longitudinal polarization is concerned, is about 10%.
- Practically, there is no difference between the predictions of the SM and ACD models for the τ lepton with longitudinal polarization.
- These two models lead to the same result for the μ lepton with the transversal polarization case.
- Up to $\hat{s} = 0.6$, the maximum difference in the predictions of these two models is about 12% for the τ lepton with the transversal polarization case.
- The normal polarization of the lepton contributes very little and the difference between predictions of the two models for this polarization can never be measured in the experiments. For the τ lepton case the difference $(P_{\rm N})_{\rm ACD} - (P_{\rm N})_{\rm SM} \approx 0.5\%$ is quite a challenging one to be measured.

From this discussion we conclude that measurement of the polarizations of the lepton is not useful for establishing the UED models.

We can now discuss the prediction of the ACD model for the forward–backward asymmetry. In Fig. 3 we show the dependence of $\mathcal{A}_{\rm FB}$ on $\hat{s}$ at four fixed values of $1/R$

and SM, for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay, when the first set of form factors are considered. For the sake of completeness, we also present in this figure the forward–backward asymmetry prediction of SM. From this figure we see that the zero position of $\mathcal{A}_{\rm FB}$ is sensitive to the compactification parameter $1/R$, similar to the $B \to K^* \ell^+ \ell^-$ decay case, and in all cases it is shifted to the left compared to that of the SM prediction. Therefore, experimental determination of the zero position of $\mathcal{A}_{\rm FB}$ can give invaluable information about new physics effects. It should be noted that the zero position of $\mathcal{A}_{\rm FB}$ is practically insensitive to the choice of form factors, and it coincides for both sets.

In conclusion, we have studied the rare $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in the ACD model with a single universal extra dimension. We investigated the sensitivity of the branching ratio, lepton polarizations and lepton forward–backward asymmetry in the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay to the compactification parameter $1/R$. We found that the branching ratio and zero position of the forward–backward asymmetry are very sensitive to the presence of the compactification parameter $1/R$ and can be useful for establishing new physics effects due to the compactification of the fifth dimension.

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